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EVALUATION OF APPROPRIATE INVERSION TECHNIQUE FOR GROUNDWATER EXPLORATION

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ABSTRACT

In this paper the data induced from lightning is synthesized using suitable inversion techniques so as to delineate imperative information regarding availability of groundwater.

KEYWORDS: *Groundwater Exploration, Problem of Earth Profiling*

Article History

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INTRODUCTION

Earth profiling is an important task being undertaken all over the world for delineating feasible sites of minerals, oil or groundwater occurrence. Experts and amateurs working in this field are practicing numerous methods for exploring sub-surface information. Delineating mineral resources, groundwater and the reduction of the risk posed by geological hazards require more and more accurate and detailed knowledge and mapping of the underground (Fancsik, T. et. al., 2021). Here, in order to extract relevant information regarding the occurrence of groundwater, lightning data is worked upon on the lines suggested by Grimm, 2002. The techniques of generalized linear inversion and Marquardt inversion are discussed with the nature of their applicability (Evans et. al., 2002, Haber, 2004) in context of the problem of earth profiling. The essence of inversion technique is that the processing and interpretation of data of complex subsurface geological structures is performed by discretization and the inversion procedure formulated for expansion coefficients. The method is found suitable for groundwater exploration as even with the introduction of relatively few expansion coefficients, a suitable resolution can be achieved so that the problem to be solved leads to an over determined inverse problem. Also, the inverse and forward modelling leads to delineating not only sites for groundwater occurrence but also thickness of sub-surface layers (Menke, 1984; Dobroka et. al., 2016)

THEORY

The fundamental principle of current propagation forms the basis of this study (Diendorfer, 2015; Ushio, et al. 2015; Li, D. et al. 2016). The lightning current induced in the earth provides valuable information regarding earth sub-surface profile. The data is incorporated to develop a forward problem to see through the sub-surface (Dweight, 1936; Goldman et. al., 2004). Establishment of a forward problem requires resistivity kernel function to be approximated as an exponential function of unknown coefficients, Koefoed, 1976.

$$K(\lambda_j) = \sum_{i=1}^{p} \sum_{j=1}^{m} f_i \, e^{-\varepsilon_i \lambda_j} \tag{1}$$

The mathematical elaboration of this kind expresses the variation of resistivity of different layers beneath the earth with respect to their extension in the downward direction. Here, f_i is the coefficient of exponential function; ε_i is the exponent parameter of kernel function and j is the number of point of observations on the earth surface. In matrix notation

$$[K]_{MX1} = [E]_{M\,X\,p}\,[F]_{p\,X\,1} \tag{2}$$

$$[E] = \begin{bmatrix} e^{-\varepsilon_1\lambda_1} & . & . & . & e^{-\varepsilon_p\lambda_1} \\ . & . & . & e^{-\varepsilon_5\lambda_2} & . \\ . & . & . & . & . \\ . & e^{-\varepsilon_2\lambda_{M-1}} & . & . & . \\ e^{-\varepsilon_1\lambda_m} & . & . & . & e^{-\varepsilon_p\lambda_m} \end{bmatrix} \tag{3}$$

The unknown coefficient matrix can be obtained as follows

$$[F]_{pX1} = [\overline{G}]_{p\,X\,M}\,[U]_{M\,X\,1} \tag{4}$$

Here $\tilde{G}$ is the generalized linear inversion (Backus et al., 1967; Snider, 1991) of function G which delimits the effect of matrix being singular and non-square dimensions.

$$G_{ji} = \frac{2r_j}{\sqrt{\varepsilon_i 2 + r_j 2}} \tag{5}$$

The objective behind employing generalized linear inversion is that, it offers a delicate solution of the non-linear system problem (Backus et al., 1967). Formulating a detailed procedure for obtaining inverse of a non-square matrix, the GLI is obtained as

$$[\overline{G}] = [G^T\,G]^{-1}G^T \tag{6}$$

In order to assess the applicability of the method a three layer model is assumed with the study area extending up to a range of 5 Km. In order to simplify the complexity of problem the study area is decomposed into a set of 12 observation points. Table 1 beneath provides computed values for equation (6) using equation (5). Coefficients of kernel function, and then finally take on values as given in Table 2.

Figure 1 show once the exponential function and its coefficients are estimated from the lightning induced potential difference data computed on the surface of earth, the next procedure starts with the evaluation of kernel function for different observation points.

Matlab tool is utilized to get a discreet view of the variation of Kernel function as obtained through computed values. It is observed that the resistivity kernel function incorporates the characteristic of in homogeneity on account of layer structure and acquires unique values for different layers and different observation points.

Table 1: Generalized Inverse of Function G

-741923	5851848	-38553470	26428766	25950092.37	16389112.96
904672.8	-7436498	48993629	-33585717	-32977347.64	-20827254.5
-2043136	1670835	-11009428	7547487.2	7410643.959	4680253.824
10463..96	-86884.64	574041.37	-393845.1	-386628.991	-244156.435
-78.8455	705.1738	-4819.072	3344.567	3274.10838	2064.81341
.69714	-6.196523	4691145	-35.883	-34.29348	-21.32920
7740066.09	1128069	-3785906	-7457099	-10240706	-23096124
-9836062.6	-1433554	4811109	9476449	13013847	29350461
2210321.103	322122.9	-1081165	-2129549	-2924464	-6595581
-115288.156	-16775.2	56437.37	1111133.3	152605.11	344129.94
972.627	138.1164	-481.9275	-945.089	-1298.24	-2917.657
-9.7681673	-.97143	5.551439	10.4183	14.10544	31.106661

Table 2

I	F(i)
1	.2857
2	.164642
3	.14285
4	.08231
5	.07142
6	.041155

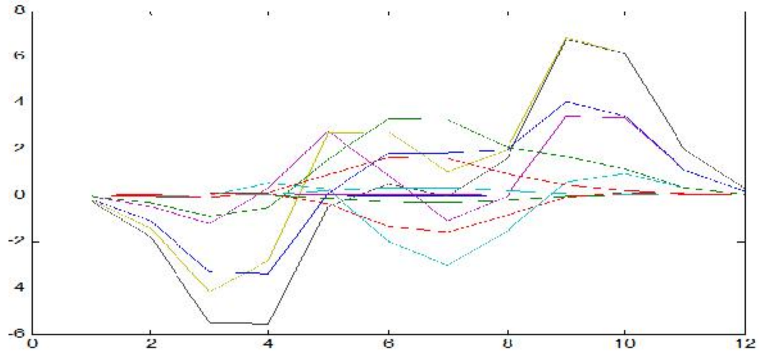

Figure 1: Resistivity Kernel Function for Observation Points.

Table 3 Shows the resistivity kernel function as estimated is utilized for the interpretation of resistivity transform function using a recurrence relation based on the work of Pekeris, 1940 and later extended by Koefoed, 1976.

$$T_L(\lambda_j) = \rho_N [2K(\lambda_j) + 1] \quad (7)$$

Table: 3: Resistivity Transform Obtained Using Lightning Data

Λ(j)	1	.5	.333	.25	.2	.1666	.14285
T(Λ)	5.860585	5.860599	5.860975	9.862659	9.866288	11.871897	11.879236

Table 4

.125	.111	.1	.05	.0333
11.888007	13.897951	13.908863	14.047349	14.204395

Simultaneously, an artificial three layer earth model is assumed with parameters

$\rho_1 = 100\,; h = 10$

$\rho_1 = 4\,; h = 30$

$\rho_3 = 20; h = \infty$

For this assumed model too resistivity transform is evaluated using the recurrence relation given by Pekeris, 1940.

$$T_{MO}(\lambda) = \frac{T'_N(\lambda) + T_{N-1}(\lambda)}{1 + \frac{T'_N(\lambda)\, T_{N-1}(\lambda)}{(\rho_{N-1})^2}} \tag{8}$$

Where,

$$T'_N(\lambda) = \rho_N (\tanh \lambda h_N) \tag{9}$$

Table 5: Resistivity Transform Obtained for Assumed Model

λ	$T_{MO}(\lambda)$
1	9.411091
.5	11.93513
.333	13.31036
.25	14.22263
.2	14.894
.1666	15.8404
.14285	15.84604
.125	16.20277
.111	16.50719
.1	16.77146
.05	18.35025
.0333	19.1764

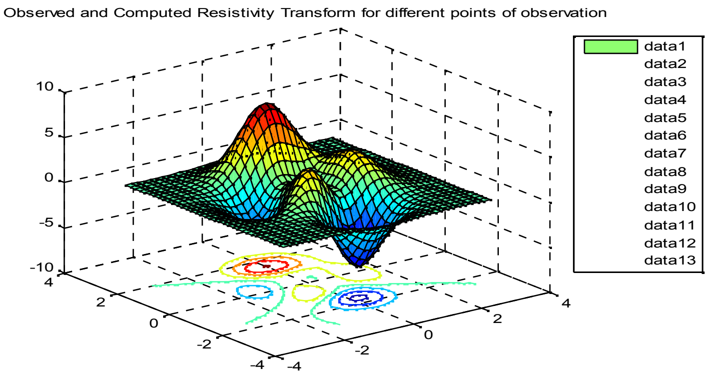

Figure 2: Resistivity Transformation.

Figure 2 shows the resistivity transforms functions as depicted above are a non-linear function of model parameters, ρ and h. A discreet and iterative inversion scheme is sought out for the complete resolution of earth data which is further refined to get closer to groundwater locations. Koefoed (1976) proposed an interactive iteration technique for bridging the gap between the assumed earth model and the computed parameter values using the formulation

$$T_L = T_{MO} + \sum (P_L - P_{MO}) \frac{\partial T_i}{\partial P_j} \tag{10}$$

$$T_L - T_{MO} = \sum (P_L - P_{MO}) \frac{\partial T_i}{\partial P_j} \tag{11}$$

With,

$$J = \frac{\partial T_i}{\partial p_j}$$

being called as the sensitivity matrix.

Also,

$$(P_L - P_{MO}) = \Delta P$$

Is considered as the correction key for each iteration step

Then the equation in matrix form would become

$$[\Delta T] = [\Delta P\ ]\quad [J] \tag{12}$$

This correction step is computed while inverting the sensitivity matrix as

$$[\Delta P] = [\tilde{J}]\,[\Delta T] \tag{13}$$

Here $\tilde{J}$ is the generalized linear inversion obtained as

$$\tilde{J} = [J^T J]^{-1} \tag{14}$$

Then for each iterative step a new predicted data set will be

$$T'_{new} = T_{MO}(\lambda) + \ \Delta P \tag{15}$$

The steps goes on this way until the assumed parameters approach the true data and thus with the final parameters is the accepted one corresponding to the observed data set.

The above discussed generalized inversion of sensitivity matrix and the parameters F and G results as a consequence of direct attempt of minimizing cumulative square of data matrix (Backus et al., 1967: Turai, E., et al., 2001; Tavakoli, S. et al., 2016; Gyulai, A., et al., 2017). An acute problem arises when the inverse of $[J^T J]$ does not exist. In such cases, an alternate route is approached by Levenberg (1944) and later by Marquardt (1963). The procedure named as the damped least square inversion is later discussed in detail by Inman, 1975.

An extensive analysis of both inversion techniques reveals that there is absolutely no difficulty with the use of ridge regression or the least square solution. However, it is observed by various authors that a perfect resolution of the assumed model is not obtained. Thus a generalized linear inversion technique has been used for the detailed analysis of the problem.

CONCLUSIONS

The present paper discusses an efficient method for the prediction of earth resistivity data utilizing the information provided by lightning phenomenon. The computation procedure is simple and straight forward with the added significance that the lightning data is directly used in the matrix method. The inversion techniques discussed here rely on the condition of whether the problem is underdetermined or over determined. MS-Excel 2007 and Matlab are customized for fast convergence of results.

REFERENCES

1. *Backus, G.E. and Gilbert, J.F., Numerical application of a formalism for geophysical inverse problems, Geophys. J. R. Astr. Soc., 13, 247-276, 1967*
2. *Diendorfer, Gerhard. A review of 25 years of lightning research in Austria from 1991-2015.World meeting on Lightning. 2016.*
3. *Dobroka, M., Norbert, P. S., Jozsef, T. and Peter, V., Interval inversion approach for an improved interpretation of well logs, Geophysics 81(2), D155-D167, 2016*
4. *Dweight, H.B., Calculation of resistances to ground, AIEE Trans., 55, 1319-1328, 1936*
5. *Evans, S., and Scales, P., Inverse problems as statistics, Inverse Problems, 18, 1-43, 2002*
6. *Fancsik, T., Turai, E., Szabó, N.P. et al. Evaluation of induced polarization measurements using a new inversion method. Acta Geod Geophys 56,623–643 (2021).*
7. *Ganse, Andrew, A., and Robert, I.O., Effects of model discretization on the statistics of model parameters in non-linear inverse problem, J. Acoust. Soc. Am., 121, 3125-3130, 2007*
8. *Goldman, M., and Neubauer, F. M., Groundwater exploration using integrated techniques, Surveys in Geophysics, 15(3), 331-361, 2004*
9. *Grimm, R.E., Low -frequency electromagnetic exploration for groundwater on mars, J. Gephys. Res., 107, 5006-5008, 2002*
10. *Gyulai Á, Szűcs P, Turai E, Baracza MK, Fejes Z, Geoelectric characterization of thermal water aquifers using 2.5D inversion of VES measurements. Surv Geophys 38(2):503–526, 2017*
11. *Haber, E., Quasi-Newton methods for large scale electromagnetic inverse problems, 21, 305-317, 2004*
12. *Inman, J. R., Resistivity inversion with ridge regression, Geophysics, 40, 798-817, 1975*
13. *Koefoed, O., Progress in the direct interpretation of resistivity soundings: an algorithm, Geophys. Prospect., (24),617-632, 1976*
14. *Levenberg, K., A method for the solution of certain non-linear problems in least squares, Quart. Appl. Math., (2), 164-168, 1944*
15. *Li D, Azadifar M, Rachidi F, et al. Analysis of lightning electromagnetic field propagation in mountainous terrain and its effects on ToA-based lightning location systems. J Geophys Res Atmos., 121(2):895–911, 2016*
16. *Marquardt, D. W., An algorithm for least squares estimation of non linear parameters, J. Soc. Ind. Appl. Math., (11), 431-441, 1963*
17. *Menke W (1984) Geophysical data analysis: discrete inverse theory. Academic Press*
18. *Pekeris, C.L., Direct method of interpretation in resistivity prospecting, Geophysics, 5, 31-46, 1940*
19. *Snider, R., An extension of Backus-Gilbert theory to non-linear inverse problems, Inverse Problems, 7, 409-433, 1991*

20. *Tavakoli S, Bauer TE, Rasmussen TM, Weihed P, Elming SA, Deep massive sulphide exploration using 2D and 3D geoelectrical and induced polarization data in Skellefte mining district, northern Sweden. Geophys Prospect 64:1602–1619, 2016*
21. *Turai E, Dobróka M, A new tool for surveying waste site, oil reservoir and oil-contaminated soil: the Generalized TAU Transform method. In: Besenyei L, Shammazov AM (eds) Intellectual service for oil & gas industry: analysis, solutions, perspectives. University of Miskolc, Miskolc, pp 156–161, 2002*
22. *Ushio T, Wu T, Yoshida S. Review of recent progress in lightning and thunderstorm detection techniques in Asia. Atmos Res. 154:89–102, 2015*